Elimination of Ringing Signals for a Lossless, Multiple-Section Transmission Line

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Abstract —**Transient processes are examined for a lossless, multiple-section transmission line terminated with resistive load. The signal components of the transmitted wave at the load end are examined in detail. The ringing effect is primarily due to two factors: (1) the voltage difference between the first arriving wave and the quasi-steady voltage and (2) the voltage signals generated by internal reflection–transmission processes. By properly choosing the values of the characteristic impedances of the signal line, we find that a multiple-section line in which the sections are of equal length can provide high-quality signals to the load. Several examples are given to illustrate this new methodology.**

I. INTRODUCTION

IN MANY APPLICATIONS of high-speed digital devices and systems, transmission media or interconnections play an important role at every level of integration. Without an appropriate arrangement of transmission media, the digital signal propagating from one position to another through interconnections suffers severe distortion. For a lossless line, signal distortion is primarily caused by a ringing effect at both the rising and falling edges of the digital signals. In principle, the ringing effect is due to the impedance discontinuities along the transmission path. When a signal is incident on a signal line, the internal reflection and transmission at the discontinuities generate signal components which successively arrive at the output and thus a ringing effect occurs. This ringing effect may cause fault logic switchings and/or degrade the slew rate of the transition edges, which introduces time skew. In particular, the ringing effect makes the performance of the transmission line dependent on the signal frequency. When the duration of the ringing signal is comparable to the pulse width, the ringing signal superimposes on the original signal to form the distorted signal at the output. Depending on the signal frequency, the superposition process can be either constructive or destructive. The work reported here is partly motivated by our desire to deliver nonskewed, nondistorted clock signals to a large number of loads through multiple-stage integrated interconnections in which the characteristic impedance of one line section may be different from those of the others. Since nonperiodic digital signals may appear in a digital system, without treating the problem in the frequency domain, it is necessary to study the transient behavior of a multiple-section transmission line.

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Transients on transmission lines represent a fundamental problem in wave guidance and radiation. Many important ideas and insights connected with functions of both time and space show up in transient phenomena. In order to understand the propagation characteristics of pulse signals in high-speed digital systems, it is therefore pertinent to study the transient processes on signal lines. Numerous authors have contributed significantly to the study of transient analysis of transmission lines [1]–[4]. Tai *et al.* [1], [2] used the Volterra integral equation and the singularity method as well as the classical method to examine transients on lossless transmission lines terminated with arbitrary loads. Gu and Kong [3] investigated the transmitted, reflected, and coupled waves for a step signal incident on single and coupled lines with capacitively loaded junctions. For lossy lines, Schutt-Aine and Mittra [4] used a combined frequency-domain, time-domain approach to study transient scattering parameters for transmission lines loaded with active devices.

Most of the work thus far has mainly focused on the formulation of transient signals but has seldom addressed methods for obtaining high-quality signals on lossless, multisection signal lines. In other words, the ringing effect and impedance matching techniques for digital signal lines have been largely ignored. In this paper, we use the time-domain approach to investigate the transient processes of a lossless, multiple-section transmission line and to study the causes of the ringing effect at the load end. We reveal that the ringing effect is primarily due to two factors: (1) the voltage difference between the first arriving wave and the quasi-steady-state voltage and (2) the voltage signals generated by internal reflection–transmission processes which successively arrive at the output after the first arriving wave. To take these considerations into account, we find that, by employing a multiple-section transmission line in which the sections are of equal length, the ringing effect due to the impedance discontinuity can be largely eliminated.

II. ANALYSIS

When a step pulse signal is incident upon a multiple-section signal line, part of the incident signal will be reflected back to the source and most of the incident energy will arrive at the output port. The incident wave coming from the source side will incur reflection and transmission processes at each of the discontinuity interfaces. The signal

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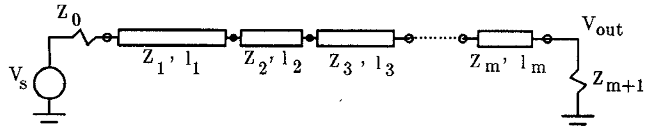

Fig. 1. An m-section transmission line.

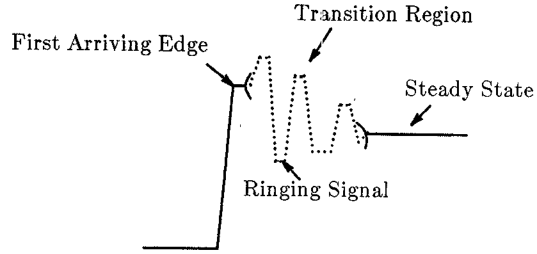

Fig. 2. Waveform at the load end for a step signal at the input.

components which undergo less propagation delay due to internal reflection–transmission processes will reach the load earlier. In the time domain, the output is the summation of all signal components which successively arrive at the load. For a lossless m-section transmission line, as shown in Fig. 1, the output at the load end is

$$V_{out}(t) = V_s \tau_{0,1}\tau_{1,2}\cdots\tau_{m,m+1}u(t-T_{line}) \cdot \left[1+\sum_{i=0}^{m-1}\sum_{j=i+1}^{m}\gamma_{j,j+1}\gamma_{i+1,i}\prod_{k=i+1}^{j-1}(\tau_{k,k+1}\tau_{k+1,k})u\left(t-2\sum_{k=i+1}^{j}T_k\right)+\cdots\right] \quad (1)$$

where

$$\tau_{0,1}=\frac{Z_1}{Z_0+Z_1} \quad (2a)$$

$$\tau_{i,i+1}=\frac{2Z_{i+1}}{Z_{i+1}+Z_i} \quad \text{for } i\geq 1 \quad (2b)$$

$$\gamma_{i,i+1}=\tau_{i,i+1}-1 \quad \text{for } i\geq 1 \quad (3)$$

$$T_i=\frac{l_i}{v_i} \quad (4a)$$

$$T_{line}=\sum_{i=1}^{m}T_i. \quad (4b)$$

V_S is the amplitude of the step voltage, $u(t)$ is the unit step function, $Z_i\,(i=1,2,\cdots,m)$ are the line impedances, l_i are the line lengths, and v_i are the signal propagation velocities. In (1)–(4), $\tau_{i,i+1}$ represents the transmission coefficient from the ith into the $(i+1)$th section; while $\gamma_{i,i+1}$ is the reflection coefficient for the signal reflected back to the ith section at the $i, i+1$ interface. The product terms of transmission coefficients are set to 1 when the lower limit is larger than the upper limit. The double summation terms designate the contribution from the signal components which undergo reflections twice before arriving at the output. Of course, higher order reflection components also contribute to $V_{out}(t)$. For simplicity, these higher order reflection terms are not included in (1). The double reflections can occur at both ends of a single line section or it may happen at two interfaces which are not connected by a single line section. Note that only signal components which undergo an even number of reflections will arrive at the output end.

In Fig. 2, we depict the possible output waveform at the load end. The output signal consists of three parts: (1) the first arriving edge, (2) the transition region, and (3) the steady-state region. The voltage magnitude of the first arriving edge may be either larger than or smaller than that of the steady-state value, which is solely determined by the line impedances $Z_i\,(i=0,1,\cdots,m+1)$. However, depending on values of line length l_i and line impedances Z_i, the voltage level in the transition region increases (decreases) monotonically or oscillates from the level of the first arriving edge to the steady-state value. When the pulse width is comparable to the duration of the transition region, the ripples in the transition region cause the distortion of signal at the load end. Furthermore, the ripple makes the performance of the signal line dependent on the pulse frequency.

A. Elimination of Ripple Due to First Arriving Wave

The step response $V_{out}(t)$ of (1) is a rather complicated function which depends on line impedances Z_i, line lengths l_i, propagation velocities v_i, and time t. As shown by the first line of the right-hand side of (1), the amplitude of the first arriving signal V_+ is a function of line impedances Z_i only. Although the variations of line lengths l_i may change the propagation delays T_i, such variations will not affect the value of the arriving edge V_+. To obtain good quality output signals, it is necessary to eliminate the voltage difference between the first arriving edge and the steady state of the output signal. For an m-section transmission line, as shown in Fig. 1, the magnitude of the first arriving edge V_+ is given by

$$V_+=V_S\frac{Z_1}{(Z_0+Z_1)}\frac{2Z_2}{(Z_1+Z_2)}\cdots\frac{2Z_{m+1}}{(Z_m+Z_{m+1})} \quad (5)$$

where Z_0 and Z_{m+1} represent the source and load impedances, respectively. The steady-state voltage V_{SS} is

$$V_{SS}=\lim_{t\to\infty}V_{out}(t)=V_S\frac{Z_{m+1}}{Z_0+Z_{m+1}}. \quad (6)$$

Note that the steady-state voltage is independent of the line impedances Z_i. At $t=\infty$, the effects of the internal multiple reflections on the output voltage vanish, and the source and load impedances serve as a voltage divider. The necessary condition for obtaining nondistorted signals at the load end is

$$V_+=V_{SS}. \quad (7)$$

As an example, for a uniform signal line terminated with a matching load, i.e., $Z_1=Z_2=\cdots=Z_m=Z_{m+1}$, (5) re-

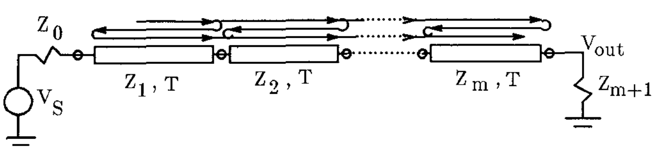

Fig. 3. An equal-length transmission line and its transmitted and reflected waves at discontinuities.

duces to

$$V_+ = V_S\frac{Z_{m+1}}{Z_0+Z_{m+1}}. \tag{8}$$

Under this condition, no impedance discontinuity exists and we get $V_+ = V_{SS}$. The voltage wave incident from the source will arrive at the output terminal without any distortion.

Without eliminating the voltage difference between the first arriving edge and the steady state, there is no way to obtain a nondistorted step output when the input is a step signal. However, the elimination of voltage difference does not imply the removal of a ringing wave within the transition region. The internal reflection signal components which arrive at the output some time after the first incident wave will add the ripple noise to $V_{out}(t)$. To ensure a good quality signal at the output, it is therefore necessary to reduce the magnitude of the ripple within the transition region.

The internal reflection processes are so involved that no physical insight can be drawn from their mathematical expressions. The effects of internal reflections on the output $V_{out}(t)$ depend on the line impedance Z_i, line lengths l_i, etc. Equations (5)–(7) impose a constraint on the values of Z_i. With the initial values of Z_i, one can simulate the transmission line performance on a computer until the optimal values of Z_i and l_i are obtained. The basis of this optimization process is to arrange the signal line in such a way that the multiple reflection–transmission signals mostly cancel out each other when they arrive at the output. Unless the length of each line section has a certain relationship, it is very difficult, if not impossible, to obtain simple analytical formulas which dictate the cancellation processes. Here we present a method to reduce the ripple wave within the transition region when each section of the signal line has the same electrical length.

B. Elimination of Ripples Due to Internal Reflections

The ripple within the transition region stems from the internal reflection–transmission process. In general, such a process is a convergent event. The signal components which undergo fewer reflections and transmissions along the signal line make a larger contribution to the ripple. Instead of dealing with all the signal components, we should therefore focus on the first few signals which arrive at the output immediately after the first arriving edge.

Fig. 3 shows an m-section equal-length transmission line. The incident signal from the left-hand side undergoes transmission and reflection processes at each of the discontinuity junctions. We assume that each line section has the same propagation delay T. The signal components which undergo two reflections at both ends of a line section will reach the output at the same time. In order to eliminate the ripple wave which arrives at the load $2T$ after the first arriving wave, we set the sum of them to 0. This gives us the following:

$$\left(\sum_{i=1}^{m}\frac{Z_{i+1}-Z_i}{Z_{i+1}+Z_i}\,\frac{Z_{i-1}-Z_i}{Z_{i-1}+Z_i}\right)\prod_{j=1}^{m}\frac{2Z_{j+1}}{Z_{j+1}+Z_j}=0. \tag{9}$$

If we apply (7) to Fig. 3, we obtain

$$\frac{Z_1}{Z_0+Z_1}\,\frac{2Z_2}{Z_1+Z_2}\,\frac{2Z_3}{Z_2+Z_3}\cdots\frac{2Z_{m+1}}{Z_m+Z_{m+1}}=\frac{Z_{m+1}}{Z_0+Z_{m+1}}. \tag{10}$$

In (9) and (10), Z_0 and Z_{m+1} represent the source and load impedances, which are known quantities. There are m unknown variables in (9) and (10), namely Z_i, $i = 1,2,\cdots,m$. Theoretically, we can arbitrarily choose the values of $m-2$ impedances and solve the remaining two impedances via (9) and (10). Therefore we do not have unique solutions to the above equations. Although we have the freedom to choose the impedance values, the whole set solution $(Z_1, Z_2,\cdots, Z_m)$ has to be physically realizable.

The above shows the procedure for eliminating the ripple due to the first reflected signal. Of course, using the same method described above, we can remove the ripple due to the higher order reflected signals. As an example, the condition for eliminating the second-order ripple, which arrives at the load $4T$ after the first arriving wave, is given by

$$\left[\sum_{i=1}^{m}\left(\frac{Z_{i+1}-Z_i}{Z_{i+1}+Z_i}\right)^2\left(\frac{Z_{i-1}-Z_i}{Z_{i-1}+Z_i}\right)^2\right.$$
$$+\sum_{i=1}^{m-1}\frac{Z_{i+2}-Z_{i+1}}{Z_{i+2}+Z_{i+1}}\,\frac{Z_{i-1}-Z_i}{Z_{i-1}+Z_i}\,\frac{2Z_{i+1}}{Z_i+Z_{i+1}}\,\frac{2Z_i}{Z_i+Z_{i+1}}$$
$$+\sum_{i=1}^{m-1}\frac{Z_{i+1}-Z_i}{Z_{i+1}+Z_i}\,\frac{Z_{i-1}-Z_i}{Z_{i-1}+Z_i}$$
$$\left.\cdot\sum_{j=i+1}^{m}\frac{Z_{j+1}-Z_j}{Z_{j+1}+Z_j}\,\frac{Z_{j-1}-Z_j}{Z_{j-1}+Z_j}\right]$$
$$\cdot\prod_{k=1}^{m}\frac{2Z_{k+1}}{Z_{k+1}+Z_k}=0. \tag{11}$$

The first term in (11) denotes the signals which bounce back and forth twice in one section of the transmission line; the second term represents the signals which bounce back and forth one time in two directly connected sections of the signal line; and the third term represents the signal components which bounce back and forth one time in two separate sections of the signal line. These three types of signals arrive at the load end at the same time. The solutions to (9)–(11) produce the values of characteristic impedances which delete the ripples caused by the first arriving edge, first- and second-order reflected waves.

However, (11) reveals that the algebraic evaluation for the elimination of the second-order reflected signal is more complicated than that of the first reflected signal. For many practical applications, the elimination of both the overshooting first arriving and the first reflected signals will result in good quality signals at the output. By choosing an equal electrical length for each section of the signal line, we avoid the involvement of time factor t in the analysis of the transient signal. Another advantage of this technique is that, since there is no constraint on the value of length, we can choose the proper line length to fit the physical layout.

It is pertinent to point out that the nondistorted output at the load end does not imply that nondistorted signals will also be obtained at other positions along the signal line. As the impedance discontinuities occur, there exist forward $(+z)$ and backward $(-z)$ traveling waves in each section of the signal line. These two opposite traveling waves cause the signal in the transmission line to become distorted. And this makes it very difficult, if not impossible, to formulate the signal construction process within the signal line without the involvement of the time factor.

III. Examples

As a first example, we consider a single-section transmission line terminated with source and load impedances at both ends. For such a simple line, (10) implies

$$\frac{Z_1}{(Z_0+Z_1)}\frac{2Z_2}{(Z_1+Z_2)}=\frac{Z_2}{Z_0+Z_2} \tag{12}$$

where Z_1 is the characteristic impedance of the line, and Z_0 and Z_2 are source and load impedances, respectively. Equation (9) is not applicable to a single-section line. The only unknown variable in (12) is Z_1 and the solution turns out to be

$$Z_1=\begin{cases} Z_0 \\ Z_2. \end{cases} \tag{13}$$

This is an obvious solution. To impose the condition that the first arriving voltage be equal to the steady-state voltage at the load, the characteristic impedance of the signal line should be equal to either the source or the load impedances. For such a transmission configuration, we obtain a nondistorted signal at the load end. If we apply (9) to a two-section line, we obtain

$$Z_0Z_2=Z_1Z_3. \tag{14}$$

Now Z_0 and Z_3 are the source and the load impedances, respectively. Combining (14) with (10) and assuming $m=2$, we obtain

$$Z_1=Z_0$$
$$Z_2=Z_3 \tag{15a}$$

or

$$Z_1=-3Z_0$$
$$Z_2=-3Z_3. \tag{15b}$$

The solution of (15a) is actually equivalent to that of (13). Apart from the propagation delay of signal lines, the waveform performance of a transmission line terminated with a matched resistive load is the same as that of the matched resistive load. Obviously, (15b) is not a valid solution to the passive circuit system.

TABLE I
Characteristic Impedances of Three-Section, Equal-Length Signal Lines

Characteristic impedances / Lines	Z_0	Z_1	Z_2	Z_3	Z_4
(a)	160	140	100	35	50
(b)	160	148	200	69	50
(c)	160	102	38	57	50

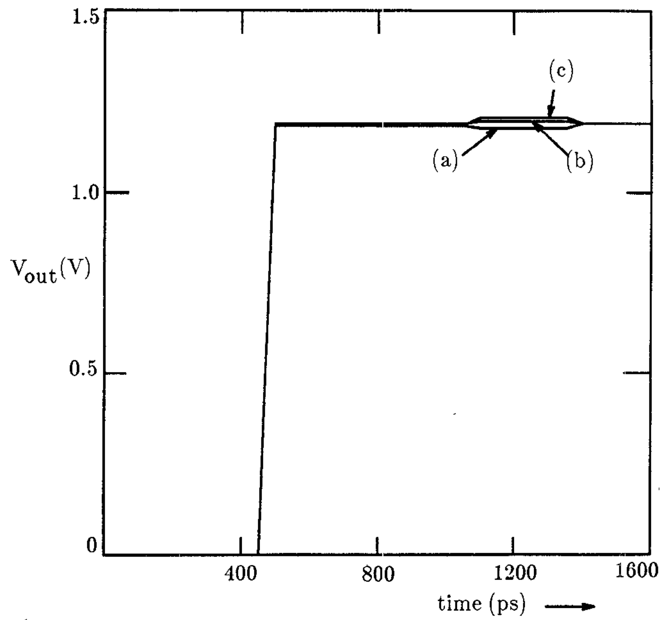

Fig. 4. Transient responses of three-section, equal-length signal lines shown in Table I.

We now proceed to a three-section equal-length transmission line. In contrast to the previous two cases, we get some illustrating results for the three-section line. For such a line, we have three unknown variables in (9) and (10). We can therefore express two of the three variables in terms of the other one. Table I shows some typical values of the characteristic impedances of a three-section line satisfying the above conditions. We assume that the source and load impedances are 160 Ω and 50 Ω, respectively. In the numerical evaluation, we choose Z_2 as a parameter and obtain the values of Z_1 and Z_3 accordingly. An examination of Table I indicates that the characteristic impedances of the line do not decrease piecewise from the source end to the load end. Fig. 4 shows the transient responses for several three-section lines. The propagation delay T for each section line is 150 ps. Although the composites of the characteristic impedances are different, the outputs are almost indistinguishable for lines (a), (b), and (c) of Table I. In this evaluation, we assume that the source has step voltage $V_S=5$ V and rise time 50 ps. Although impedance discontinuities exist along the transmission path, the step output waveform is almost nondistorted. The ripples at $t=1050$ ps are due to second-order transmission–reflection signals on the signal line. After $t=1350$ ps, the outputs become steady state for the three

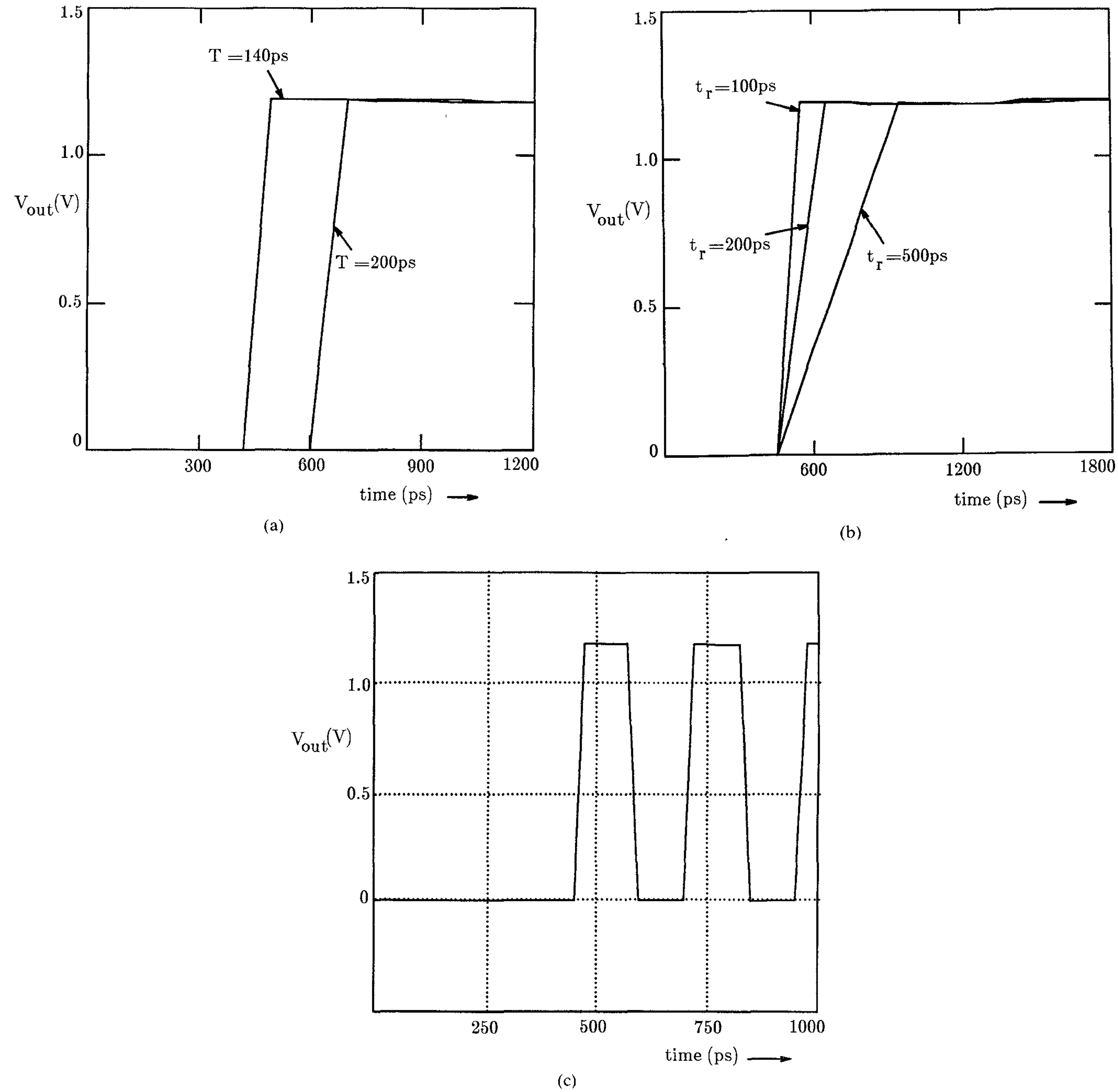

Fig. 5. Transient behavior of a three-section, equal-length line. (a) Step responses for different propagation delays. (b) Step responses for different rise time of signals. (c) Responses to a 4 GHz signal. In all cases, $Z_0=160$, $Z_1=140$, $Z_2=100$, $Z_3=35$, and $Z_4=50$.

lines in Table I, i.e., third-order and higher order transmission–reflection signals have no effect or negligible effect on the output signals. By deleting the ripple due to the first arriving and first reflected waves, we can thus obtain good quality signals at the output.

Fig. 5(a) shows the outputs for different propagation delays of the signal line, where T is the propagation delay of each section. It is clear that the performance of such a signal line is independent of the physical length of the line as long as each section of the signal line has the same electrical length. Fig. 5(b) shows the step responses of a three-section line for different rise times t_r of the input signals. The internal reflection–transmission process on the line has little effect on the integrity of signal rise edge. Of course, we can use the superposition principle to construct the output waveform when the input is a pulse train. Fig. 5(c) shows the simulation result for a 4 GHz digital signal having 25 ps rising time and 100 ps pulse width. In Fig. 5(b) and (c), the propagation delay of each line segment is 150 ps. So far we have concentrated mainly on the transient responses at the load end. It is illustrative to show the transient behavior at positions within the transmission line. The transients at respective discontinuities are shown schematically in Fig. 6. It appears that the

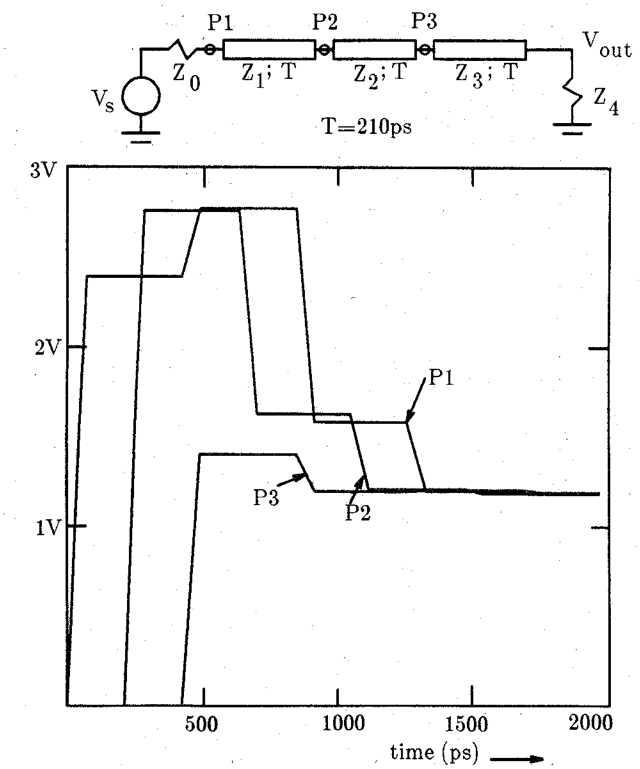

Fig. 6. Transient responses at respective positions in line (b) of Table I.

waveshapes exhibit rather big oscillations from the first arriving edge to steady state. The signal at point $P3$ reaches the steady state faster than those at positions $P2$ and $P1$. If we assume the propagation delay of each section is T, a close examination indicates that the transients reach the steady-state value at $4T$, $5T$, and $6T$ for positions $P3$, $P2$, and $P1$ respectively. Generally, the voltage at z changes to steady-state value after $t = 3T + (L - z)/v_i$, where L is the total physical length of the signal line, z is the distance measured from the source to the observation point, and $v_i (i = 1,2,3)$ are the propagation velocities of the signal in the respective regions. After $6T$, every position in the signal line has the same electric potential and the internal multiple transmission–reflection processes subside. We should mention that all the simulations of transient waves shown above were carried out by using the SPICE simulator.

These examples illustrate the novel method for obtaining almost nondistorted transient signals at the load end of lossless, multiple-section transmission lines. This technique is different from the quarter-wavelength transformer [5], which allows the existence of transient ripple at the load end. If we take the derivative of the left-hand side in (12) with respect to Z_1 and set the equation to zero, we obtain

$$Z_1 = (Z_0 Z_2)^{1/2}. \tag{16}$$

For a single-section transmission line, (16) is the condition for obtaining maximum first arriving wave at the load end. It is also the requirement of the characteristic impedances for a quarter-wavelength transformer. If (16) holds, there exists transient ripple at the output when the input is a step function voltage. The transient ripple causes the dependence of the transmission line on the signal frequency.[1] However, when the propagation delay of the transmission line is one quarter of one cycle period of the input signal, we get the maximum output voltage at that particular frequency.

IV. Conclusions

The above detailed analysis of transient phenomena on lossless, equal-length transmission lines has shown that the ringing effect of the transmitted signal at the load end can be largely eliminated if we properly choose the values of characteristic impedances. The single-section transmission line terminated with matched loads turns out to be a special case of the above general methodology. By deleting the voltage difference between the first arriving wave and the quasi-steady-state voltage and canceling the internal reflection–transmission signals, a multiple-section equal-length transmission line can provide a good quality signal to the load. The method developed here can be applied to multiple-stage integrated interconnections.

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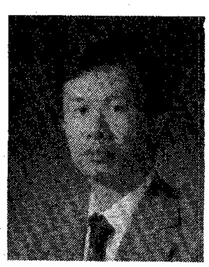

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[1] One of the reviewers pointed out that the use of quarter-wave transformers results in a bandpass characteristic. It may indeed be possible to obtain a high-pass characteristic by using a nonuniform section of line.